1. Details of module and its structure

Module Detail	
Subject Name	Physics
Course Name	Physics 01 (Physics-Part 1, Class XI)
Module Name/Title	Unit 2, Module 9, **Problems -Motion in two dimension** Chapter 4, Motion in a plane
Module Id	Keph_10404_eContent
Pre-requisites	Equations of motion, projectile motion , motion in a circle.,
Objectives	**After going through this module, the learners will be able to**: Apply equations of motion for solving problems in two dimension
Keywords	Projectile motion problem, circular motion problems,

2. Development Team

Role	Name	Affiliation
National MOOC Coordinator (NMC)	Prof. Amarendra P. Behera	Central Institute of Educational Technology, NCERT, New Delhi
Programme Coordinator	Dr. Mohd Mamur Ali	Central Institute of Educational Technology, NCERT, New Delhi
Course Coordinator / PI	Anuradha Mathur	Central Institute of Educational Technology, NCERT, New Delhi
Subject Matter Expert (SME)	Anuradha Mathur	Central Institute of Educational Technology, NCERT, New Delhi
Review Team	Prof. V. B. Bhatia (Retd.) Associate Prof. N.K. Sehgal (Retd.) Prof. B. K. Sharma (Retd.)	Delhi University Delhi University DESM, NCERT, New Delhi

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1. UNIT SYLLABUS

Chapter 3: Motion in a straight line

Frame of reference, motion, position –time graph Speed and velocity
Elementary concepts of differentiation and integration for describing motion, uniform and non-uniform motion, average speed and instantaneous velocity, uniformly accelerated motion, velocity –time and position time graphs relations for uniformly accelerated motion - equations of motion (graphical method).

Chapter 4: Motion in a plane

Scalar and vector quantities, position and displacement vectors, general vectors and their notations, multiplication of vectors by a real number , addition and subtraction of vectors, relative velocity, unit vector, resolution of a vector in a plane, rectangular components, scalar and vector product of vectors
Motion in a plane, cases of uniform velocity and uniform acceleration projectile motion uniform circular motion.

2. MODULE-WISE DISTRIBUTION OF UNIT SYLLABUS 10 Modules

The above unit is divided into 10 modules for better understanding.

Module 1	• Introduction to moving objects • Frame of reference, • limitations of our study • Treating bodies as point objects
Module 2	• Motion as change of position with time • Distance travelled unit of measurement • Displacement negative, zero and positive • Difference between distance travelled and displacement • Describing motion by position time and displacement time graphs
Module 3	• Rate of change of position • Speed • Velocity • Zero , negative and positive velocity • Unit of velocity • Uniform and non-uniform motion • Average speed • Instantaneous velocity • Velocity time graphs • Relating position time and velocity time graphs
Module 4	• Accelerated motion • Rate of change of speed, velocity • Derivation of Equations of motion
Module 5	• Application of equations of motion • Graphical representation of motion • Numerical

Module 6	• Vectors • Vectors and physical quantities • Vector algebra • Relative velocity • Problems
Module 7	• Motion in a plane • Using vectors to understand motion in 2 dimensions' projectiles • Projectiles as special case of 2 D motion • Constant acceleration due to gravity in the vertical direction zero acceleration in the horizontal direction • Derivation of equations relating horizontal range vertical range velocity of projection angle of projection
Module 8	• Circular motion • Uniform circular motion • Constant speed yet accelerating • Derivation of $a = \frac{v^2}{r}$ $or\ \omega^2 r$ • direction of acceleration • If the speed is not constant? • Net acceleration
Module 9	• Numerical problems on motion in two dimensions • Projectile problems
Module 10	• Differentiation and integration • Using logarithm tables

MODULE 9

3. INTRODUCTION

This module is dedicated to solving problems in 2 dimensions. As we have learnt in earlier modules motion in one and two dimensions have similar ideas about position, velocity and acceleration, but they are a little more complex in 2 dimension as compared to one dimension.

Many situations demand a clear understanding of the problem before we set to calculate. Physics is never about choosing a formula and just plugging in the values and using mathematical rules to get the solution.

Physics problems begin as word problem and terminate as mathematical exercise.
Some commonly observed habits for problem solving in physics are

Reading and Visualizing

- Read carefully to visualize the physical situation
- Translate written words into mathematical variable by an informative sketch or diagram depicting the situation.

Organization of known and Unknown Information

- Write down the quantitative information with its proper units and symbol.
- Think critically and apply physics for solving unknown quantity.

Identification of appropriate formulae and operations

- Once a strategy has been plotted for solving problem, list the appropriate mathematical formulae on their paper.
- Rearrange the formulae such that unknown quantity appears by itself on left side of the equation.
- Substitute known information in identified formulae to solve for unknown quantity.

Hence physics problems require careful reading, good visualization skills, background physics knowledge, analytical thoughts, inspection and strategy planning.

Let us collect the formulae and equations that we have learnt in this unit.

Remember if acceleration a is constant

i) $v = u + at$

ii) $v^2 = u^2 + 2aS$

iii) $s = ut + \frac{1}{2}at^2$

iv) **Maximum Height attained by Projectile is:**

$H = \frac{u^2 \sin^2\theta}{2g}$

(v) Horizontal Range:

$$R = \frac{u^2 sin\theta}{g}$$

(vi) Maximum Range:

$$R_{max} = \frac{u^2}{g}$$

THINK ABOUT THESE

Why? How? Would the equations change in case acceleration is not constant?

Why would variable acceleration problems need calculus or different mathematical operations?

4. ILLUSTRATIONS

EXAMPLE:

From the top of a tower 100 m in height a ball is dropped and at the same time another ball is projected vertically upwards from the ground with a velocity of 25 m/s. Find when and where the two balls will meet. ($g = 9.8$ m/s^2).

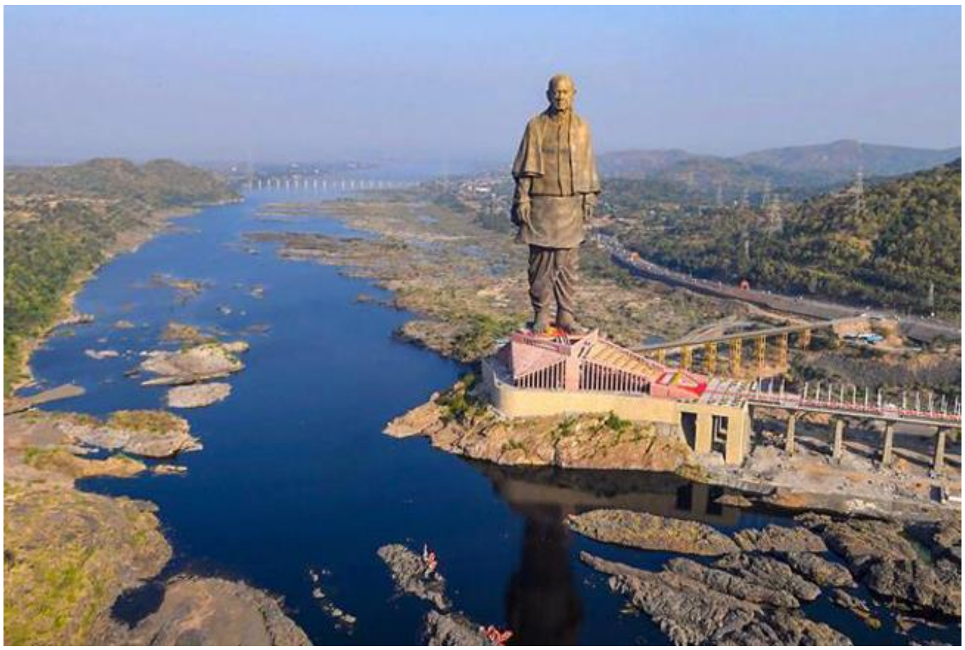
Before you proceed imagine the situation in the question. What does the word ‘dropped’ mean?

It refers to initial velocity zero because the other ball is project with a velocity of 25m/s. If the initial velocity of the second ball was also zero, it would not go against gravity, rather not go up at all.

Now also think if the second ball was not projected vertically but at an angle?

When we say the balls 'will meet' do we expect a collision? Or it could be that they pass each other? This would be the case if the line of travel is different.

SOLUTION:



Statue of Unity

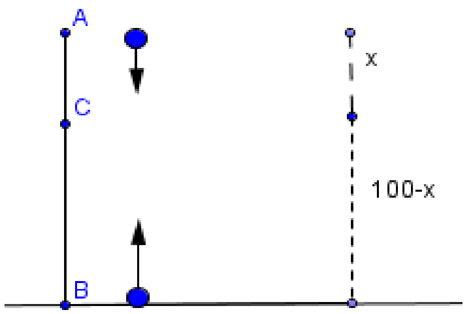
Visitors can go up to 120 m, the above question can be imagined in such a situation keeping security in mind.

Let A be the top of the 100 m tower and B be its foot.

Let the two balls meet at C after time t and

If AC = x, then BC = 100 –x,

as shown in fig.



Case 1:

Taking vertical downward motion of the ball dropped from the top, we have

Initial velocity u = 0

Acceleration a = - 9.8 m/s^2

We have chosen -9.8 ms^{-2} using Cartesian frame with (0, 0) at A

S = x

Time = t

As, S = u t + $\frac{1}{2}$ at^2

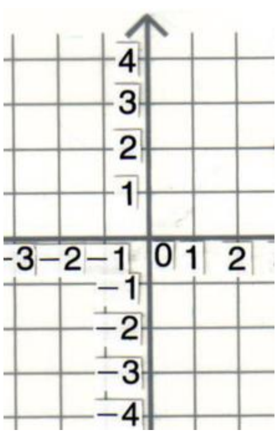
Therefore,

$$x = 0 + \frac{1}{2}(-9.8ms^{-2})t^2$$

x = - 4.9 t^2…………………………..(i)

What does the negative sign indicate?

That x is in the negative y direction



Case 2:

Taking vertical upward motion of the ball thrown up from B, we have

Initial velocity u = + 25 ms^{-1}

Acceleration a = - 9.8 ms^{-2}

S = (100 -x)

Time = t

As $s = ut + \frac{1}{2}at^2$

$(100 - x) = 25t + \frac{1}{2}(-9.8)t^2$

$(100 - x) = 25t + (-4.9)t^2$..........(ii)

By adding (i) and (ii) eq., we get

100 = 25 t

Or

t = 4s.....................................(iii)

By using (iii) in (i)

x = - 78.4 m

Hence, the two balls will meet after 4 seconds at a distance 78.4 m below the top of the tower.

This is interesting as the acceleration of the two balls were the same, yet they do not meet at the midpoint of 100 m or at 50 m or travel the same distance. According to the calculation ball which started with zero velocity travels 78.4 m while the one that started with a velocity of 25 ms^{-1} travels $(100 - 78.4 = 21.6m)$

Example

A ball thrown vertically upwards with a speed of 19.6ms^{-1} from the top of a building it reaches the ground below after 6 s. Find the height of the building.

SOLUTION

Taking the point of projection as (0, 0)

Initial velocity = + 19.6 ms^{-1}

Acceleration = g = -9.8ms^{-2}

Time of flight t = 6s

Net displacement = height of the building = say - h

Why negative sign?

Using

$$s = ut + \frac{1}{2}at^2$$

With signs we get

$$-h = 19.6 \times 6 + \frac{1}{2} \times (-9.8) \times 6^2$$

$$= 117.6 - 176.4 = -58.8$$

Height of the building = 58.8m

THINK ABOUT THIS

One equation keeping the direction in mind was sufficient to solve the problem. You could of course breakup the motion into parts and get the same result but this is easier and more elegant.

EXAMPLE

A ball is thrown vertically upwards with a velocity of 20 ms^{-1} from top of a multistory building. The height of this point above the garden on the ground is 25 m.

- **i)** **How high will the ball rise?**
- **ii)** **How long will it take to reach the garden below?**

  Take g =10 ms^{-2}

SOLUTION

https://jooinn.com/multi-storey-building.html

Taking the point of projection as (0, 0)

Initial velocity = + 20 ms^{-1}

Acceleration = g = should be -9.8ms^{-2} and one should take this value, unless given otherwise as in this case 10 ms^{-2}

Time of flight t =?

Net displacement = height of the building = say – 25m

i) Say the ball rises to a height of h m before it starts to fall again.

The highest point to which the ball rises is till its velocity =0

$$v^2 - u^2 = 2as$$

$$0^2 - 20^2 = 2(-10)h$$

$$or\ \mathbf{h = +25m}$$

ii) To determine the time of flight

$$s = ut + \frac{1}{2}at^2$$

$$-25 = 20t + \frac{1}{2}(-10)t^2$$

$$\mathbf{5t^2 - 20t - 25 = 0}$$

Or

$$\mathbf{t^2 - 4t - 5 = 0}$$

Solution to the polynomial is

$(t+1)(t-5)=0$

Or $t=-1s$ *or* $t=5s$

The first solution is not possible as mathematically it suggest that time t is negative, which it cannot be, so t = 5 s is the correct value.

EXAMPLE

Two ends of a train moving with a constant acceleration pass a certain point with velocities u and v.

Show that the velocity with which the middle point of the train passes the same point is

$$\sqrt{\frac{(u^2+v^2)}{2}}$$

SOLUTION:

https://kn.wikipedia.org/wiki/%E0%B2%9A%E0%B2%BF%E0%B2%A4%E0%B3%8D%E0%B2%B0:WAP-7_class_electric_locomotive_of_Indian_Railways.jpg

Just for imagination

Let 'x' be the total length of the train, V be the velocity of the train while passing a certain **middle point** and 'a' be the uniform acceleration of the train.

Taking the motion of the train when middle point is passing from the **given point**, we have

u = initial velocity

v = final velocity

S = x / 2

a = acceleration

Using,

$v^2 = u^2 + 2aS$

We have, $v^2 = u^2 + 2\frac{ax}{2}$ = u^2 + ax…….(i)

Taking the motion of train when the last end of train is passing from the given point, then

S = x

Now, we have,

$v^2 = u^2 + 2ax$

Or,

$ax = v^2 - \frac{u^2}{2}$ …….. (ii)

Using (ii) in (i), we get

$$v^2 = u^2 + \frac{v^2}{2} - \frac{u^2}{2}$$

$= \frac{u^2+v^2}{2}$

$\boldsymbol{v} = \sqrt{\frac{(\mathbf{u^2+v^2})}{\mathbf{2}}}$

Hence proved

EXAMPLE

A water fountain on the ground sprinkles water all around it. If the speed of water coming out of the fountain is 'v', then calculate the total area around the fountain that gets wet.

SOLUTION:

The area getting wet will be the maximum range of the water jets acting as projectiles. This helps architects to design water bowls for beautification fountains that we see with buildings, parks shopping centers etc.

Here, Maximum range of water coming out of the fountain is:

$R_{max} = v^2 / g$ …… (i)

Total area around the fountain that gets wet is:

$total\ area = \pi R_{max}^2$........ (ii)

Total area around the fountain that gets wet is:

By Using (i) in (ii), we get

$$total\ area = \pi \frac{v^4}{g^2}$$

This is the total area around the fountain that gets wet.

EXAMPLE

How high above the ground, can a girl throw a ball, if she is able to thrown the same ball up to maximum horizontal distance of 60 m.

SOLUTION:

Here, R_{max} = 60 m

Step 1:

$R_{max} = \frac{u^2}{g}$

Therefore,

$\frac{u^2}{g} = 60$ ……….. (i)

Step 2:

Let h be the maximum height attained by the ball when thrown with a speed u.

Using:

$$v^2 - u^2 = 2as$$

Here, v = 0 (at the highest point)

And a = - g

and

S = h

Therefore,

$$-u^2 = -2\,g\,h$$

Or

$$h = \frac{1}{2}\left(\frac{u^2}{g}\right)$$

Using equation (i), we get

$$\mathbf{h = \frac{1}{2}(60m) = 30m}$$

EXAMPLE

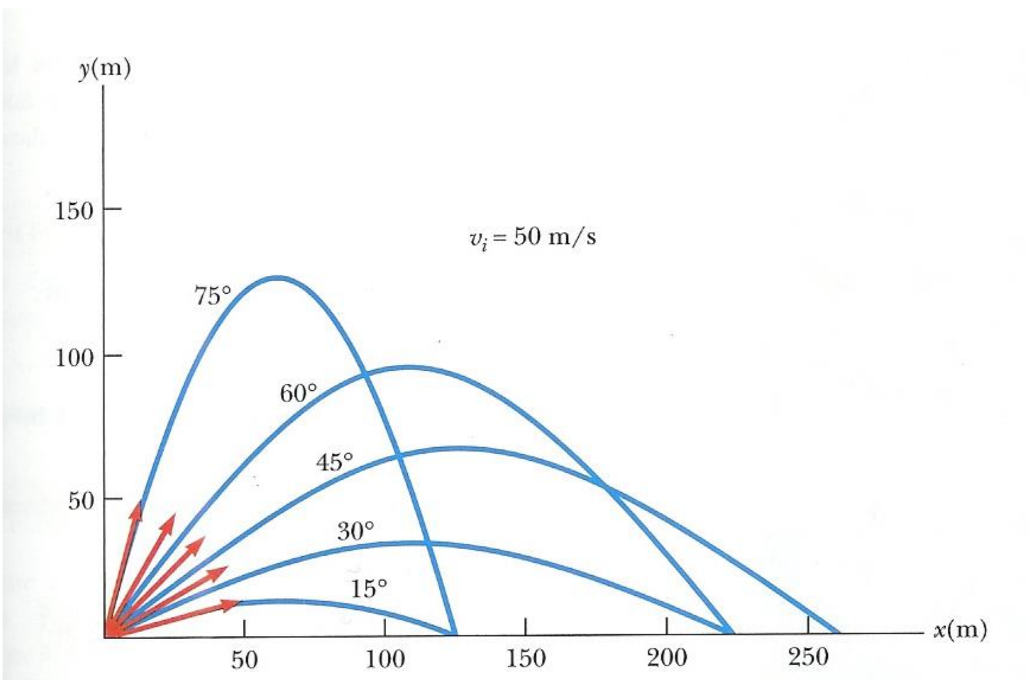
Show that the projection angle θ_0 for a projectile launched from a point is given by:

$$\boldsymbol{\theta_0 = tan^{-1}\left(\frac{4H}{R}\right)}$$

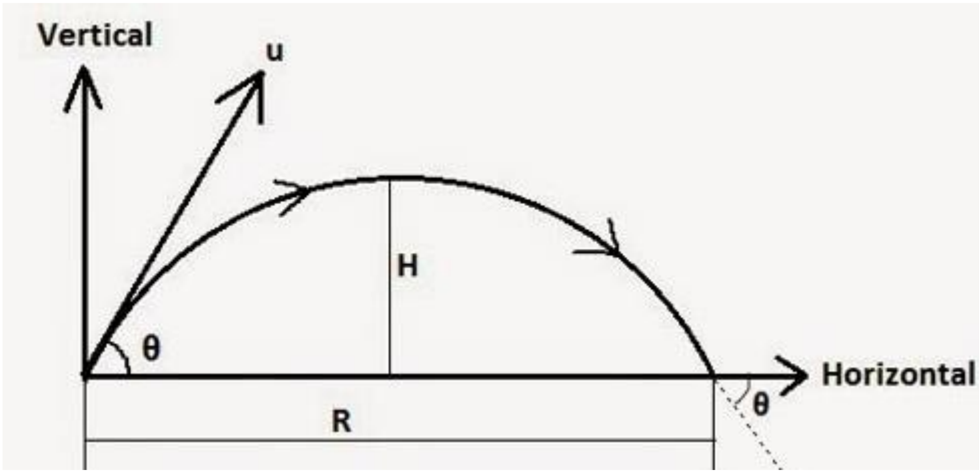
SOLUTION:

The question suggests whatever be the angle of projection of a projectile, there is a relation between the horizontal range and the maximum vertical height it can reach. The initial velocity for each case remains the same.

The path followed by projectile projected at an angle Θ_0 with velocity is shown in figure:



fden-2.phys.uaf.edu/211_fall2004.web.dir/Ryan_Boothe/ThePhysicsOfaJumpPg3.html



The maximum height H or vertical range attained by the projectile is given by:

$H = \frac{u^2 \sin^2\theta}{2g}$ …… (i)

The horizontal range of the projectile is given by:

$R = \frac{u^2 \sin 2\theta}{g} = \frac{2u^2 \sin\theta\cos\theta}{g}$ ….. (ii)

Dividing equation (i) by equation (ii), we get

$$\tan\theta = \frac{4H}{R}$$

Or

$$\boldsymbol{\theta = \tan^{-1}\left(\frac{4H}{R}\right)}$$

Hence, proved.

EXAMPLE

A boy playing on the roof of a 20 m high building throws a ball with a speed of 10 m/s at an angle of 30^0 with the horizontal.

How far from the throwing point will the ball be at the height of 20 m from the ground? (g = 10 ms^{-2})

SOLUTION:

When the ball will be at the height of 20 m from the ground i.e., the height of the point of projection, the ball travels a distance equal to the range of the ball.

Range R

$$R = \frac{u^2 sin2\theta}{g}$$

Here, u = 10 m/s

$\theta = 30^0$

H = 20 m

And

g = 10 ms^{-2}

Therefore,

$$\mathrm{R} = \frac{10^2 \sin 60^0}{10} = 100 \times \frac{\sqrt{3}}{2} \times 10 = \sqrt{3 \times 5} = 1.73 \times 5 = 8.65\ \mathrm{m}$$

EXAMPLE

A bullet fired at an angle of 45^0 with the horizontal, hits the ground 2 km away. Can we hit a target at a distance of 5 km by adjusting its angle of projection?

SOLUTION:

Here,

R = 2 km = 2000 m

$\Theta = 45^0$

Using,

$$\mathbf{R} = \frac{\mathbf{u^2 sin2\theta}}{\mathbf{g}}$$

$$2000 = \frac{u^2 \sin 90°}{g}$$

Or,

$$\frac{u^2}{g} = 2000 = 2\ km$$

Therefore,

Maximum range,

$$R = \frac{u^2}{g} = 2\ km$$

Hence, the bullet can never hit the target which is 5 km because its maximum range is 2 km. if it is fired with the same initial speed even if the angle of projection is adjusted.

EXAMPLE

Galileo, in his book 'Two new sciences', stated that "for elevations which exceed or fall short of 45° by equal amounts, the ranges are equal". Prove this statement.

SOLUTION

For a projectile launched with initial velocity v at an angle q_o, the range is given by

$$R = \frac{u^2 \sin 2\theta}{g}$$

Now, for angles, (45° + a) and (45° – a), 2 θ is (90° + 2a) and (90° – 2a) , respectively. The values of sin (90° + 2a) and sin (90° – 2a) are the same, equal to that of cos 2a. Therefore, ranges are equal for elevations which exceed or fall short of 45° by equal amounts a

EXAMPLE

A hiker stands on the edge of a cliff 490 m above the ground and throws a stone horizontally with an initial speed of 15 m s^{-1}. Neglecting air resistance, find the time taken by the stone to reach the ground/sea, and the speed with which it hits the ground/sea.
(Take g = 9.8 m s^{-2}).

SOLUTION

Varkala Beach, cliffs are found adjacent to the Arabian Sea. Varkala is well connected by road to various cities. The nearest railway station is Varkala, about 3 km away. The nearest airport is Thiruvananthapuram International Airport, about 57 km away

https://www.holidayiq.com/blog/5-dramatic-sea-cliffs-india-364.html

We choose the origin of the *x*-, and y axis at the edge of the cliff and *t* = 0 s at the instant the stone is thrown. Choose the positive direction of *x*-axis to be along the initial velocity and the positive direction of *y*-axis to be the vertically upward direction.

The *x*-, and *y* components of the motion can be treated independently. The equations of motion are:

$$x(t) = x(0) + v_x(0)\,t$$

$$y(t) = y(0) + v_y(0)\,t + \frac{1}{2}a_y t^2$$

$$x(0) = y(0) = 0$$

$$a_y = -9.8\text{ms}^{-2}$$

$$v_x(0) = 15\text{ms}^{-1}$$

The stone hits the ground when $y(t) = -490\text{m}$

$$-490\text{m} = \frac{1}{2}(-9.8)t^2$$

This gives t =10 s

The velocity components are $v_x(0) = v_x(t)$

$$y(t) = v_y(0) - gt$$

So that when the stone hits the ground

$$v_x(10) = 15\text{ms}^{-1}$$

$$v_y(10) = 0 - 9.8 \times 10 = 98\text{ms}^{-1}$$

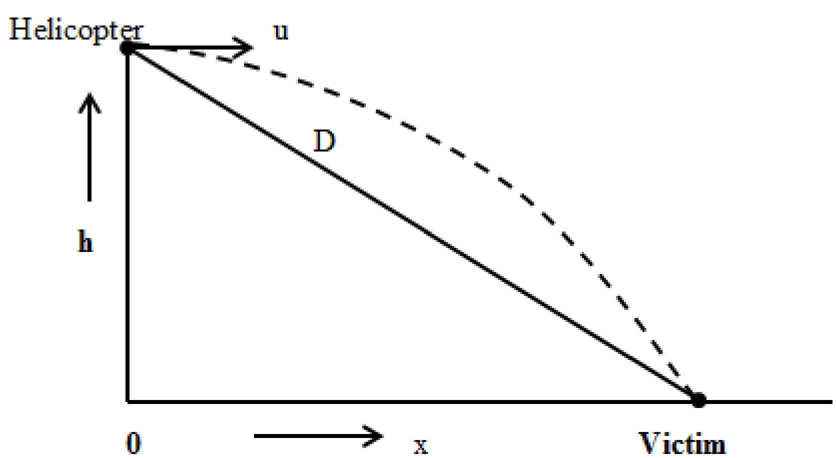
Therefore the speed of the stone is

$$\sqrt{v_x^2 + v_y^2} = \sqrt{15^2 + 98^2} = \mathbf{99ms^{-1}}$$

EXAMPLE

A helicopter on a flood relief mission flying horizontally with a speed 20 m s^{-1} at an altitude of 500 m, has to drop a food packet for a flood victim standing on an island. At what distance from the victim should the food packet be dropped?

SOLUTION



$$h = 0 + \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2h}{g}}$$

Horizontal distance covered by the food = x = $20\sqrt{\frac{2\times500}{10}} = 200m$

Distance of victim from the point of projection of food packet =

$$\sqrt{500^2 + 200^2} = 5.38 \times 100 = 538m$$

THINK ABOUT THIS

- **What if the helicopter was traveling in the opposite direction?**
- **What would be the food packet trajectory?**

EXAMPLE

A relief packet is released from a helicopter which is ascending steadily at 2 ms^{-1}.

After 2 s

i) What is the velocity of the packet?

ii) How far is it below the helicopter?

SOLUTION

https://defenceaviationpost.com/flood-situation-grim-in-ganjam-indian-navy-to-deploy-two-choppers-for-relief-and-rescue-operations/

i) Initial velocity of the relief packet is the same as the helicopter as it was traveling in it.

u = +2 ms^{-1}

Acceleration = -9.8ms^{-2}

So to find the velocity of the packet after t = 2 s

We use

$$v = u + at$$

Or, $\mathbf{v = 2 - 9.8 \times 2 = -17.6 ms^{-1}}$

What does the negative sign tell us?

The packet is moving vertically downwards. Notice it moves with a much larger speed.

ii) Distance covered by the packet will be

$$s = ut + \frac{1}{2}at^2$$

$$s = 2 \times 2 - \frac{1}{2} \times 9.8 \times 2^2$$

$$\mathbf{s = 4 - 19.6 = -15.6m}$$

Thus the packet falls through a distance of 15.6 m in 2 s but the helicopter in the same time ascends $2 \times 2 = 4$ m upwards

So the distance between the packet and the helicopter will be

$\mathbf{15.6m + 4m = 19.6m}$

EXAMPLE

A cricket ball is thrown at a speed of 28 m s^{-1} in a direction 30° above the horizontal. Calculate (a) the maximum height, (b) the time taken by the ball to return to the same level, and (c) the distance from the thrower to the point where the ball returns to the same level.

SOLUTION

a) **the maximum height is given by**

$$H = \frac{u^2\sin^2\theta}{2g} = \frac{(28\sin30)^2}{2\times9.8} = \frac{14\times14}{2\times9.8} = 10.0\text{m}$$

b) the time taken to return to the same level is

$$T = \frac{2u_0 sin\theta}{g} = \frac{2 \times 28 \times sin30}{9.8} = \frac{28}{9.8} = 2.9s$$

c) the distance from the fielder to the point where the ball returns to the same level is

$$\text{R} = \frac{(\text{u}^2\sin2\theta)}{\text{g}} = \frac{28 \times 28 \times \sin60}{9.8} = 69\text{m}$$

EXAMPLE

A parachutist bails out from an aeroplane and after dropping through a distance of 40 m, he opens the parachute and decelerates at 2 ms^{-2}. If he reaches the ground

with speed of $2ms^{-1}$, how long is he in the air? At what height did he bail out from the plane?

SOLUTION:

https://www.jbsa.mil/News/Photos/igphoto/2001310988/

https://en.wikipedia.org/wiki/Parachuting

When the parachutist falls freely,

$u = 0$,

$g = 9.8\ ms^{-2}$,
$s = 40$ m,
$t = ?$,
$v = ?$
As

$$s = ut + \frac{1}{2}at^2$$

$$\therefore \mathbf{40 = 0} + \frac{1}{2} \times 9.8 \times t^2$$

Or $$t = \sqrt{\frac{\mathbf{80}}{\mathbf{9.8}}} = \frac{\mathbf{20}}{\mathbf{7}} = \mathbf{2.86\ s}$$

Also, $\boldsymbol{v = u + gt = 0 + 9.8 \times 2.86 = 28\ ms^{-1}}$

When the parachutist decelerates uniformly:

$u = 28\ ms^{-1}$, $a = -2\ ms^{-2}$, $s = 40$ m, $v = 2\ ms^{-1}$
Time taken,

$$t = \frac{v-u}{a} = \frac{2-28}{-2} = 13\ s$$

Distance,

$$s = ut + \frac{1}{2}at^2 = 28 \times 13 + \frac{1}{2} \times 2 \times (13)^2$$

$$= 364 - 169 = \mathbf{195m}$$

Total time of parachutist in air = 2.86 + 13 = 15.86 s

Height at which parachutist bails out = 40 + 195 = 235 m

REMEMBER

The initial velocity of falling body is not always zero and one must understand it for each situation.

5. SUMMARY:

Following formulas are used to solve Numericals:

(1) $v^2 = u^2 + 2aS$

(2) $s = ut + \frac{1}{2}at^2$

(3) Maximum Height attained by Projectile is:

$$H = \frac{u^2 sin^2\theta}{2g}$$

(4) Horizontal Range:

$$R = \frac{u^2 \sin 2\Theta}{g}$$

(5) Maximum Range: $R_{max} = \frac{u^2}{g}$